B.Math I. Final Exam. PROBABILITY.
December 25. 9:30 AM - 12:30 PM.

Answer any eight of the nine problems. All nine problems carry equal weight.

1 The diameter X of a ball bearing is normally distributed with mean μ and standard deviation 0.5.

If the (absolute) deviation between the diameter X and its mean is less than 0.6 the bearing can be sold for a profit of $Rs50$. If the indicated deviation is greater than 1.2 the bearing must be discarded at a loss of $Rs100$. Otherwise the bearing may be reworked and sold at a profit of $Rs25$. Find the expected value for the profit. (Your answer has to be a numerical value.)

2 The time (in seconds) to react by an individual to a certain medicine is a random variable with *pdf* given by

$$f(x) = \begin{cases} c.x^{-2} & \text{if } 1 \leq x \leq 3. \\ 0 & \text{otherwise.} \end{cases}$$

where c is a constant. If the individual takes more than 1.5 seconds to react a light comes on and stays on until the time of reaction or until one second has elapsed which ever is less. If Y is the length of the time the light is on, find the expected value of Y.

3 If 200 numbers are chosen at random over the interval $[1, 2]$,

(a). Use the appropriate approximation to find the approximate probability that at least two of them will be less than $\frac{101}{100}$.

(b). Use the appropriate approximation to find the approximate probability that at most 90 of them will be more than 1.6.

If you make any assumptions please state them.

4. The number of years that a washing machine functions is a random variable T whose hazard rate function is given by

$$\lambda(t) = \begin{cases} 0.3 & \text{if } 0 < t < 4 \\ 0.3 + 0.5(t-4) & \text{if } 4 \leq t < 6 \\ 1.3 & \text{if } t \geq 6 \end{cases}$$

(a). If the machine is still working seven years after being purchased, find the conditional probability that it will fail within the succeeding two years.

(b). Find the *pdf* of T.

5 The number of televisions sold per day in an electronic store is a random variable X with possible values $0, 1, 2, 3, 4$ with respective probabilities

$0.05, 0.1, 0.2.5, 0.4, 0.2$. From the past experience it is known that whenever a television is purchased, a television stand is also purchased simultaneously with probability 0.4.

If Y is the number of television stands that are sold along with the televisions sold per day, find the joint probability mass function of X and Y.

Find the conditional variance of X given $Y = 1$.

6 Suppose that X and Y have the joint *pdf* given by

$$f(x,y) = \begin{cases} c & \text{if } -1 \le x \le 1,\ y \ge 0,\ x^2 + y^2 \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

where c is a constant. Find $E[X \mid Y = y]$ and $E[Y \mid X = x]$. (You might want to use the aid of a graph appropriately.)

7 A man and a woman decide to meet at a restaurant around 6 pm. for a dinner. The arrival time X (measured in hrs) of the man to the restaurant is a random variable having the *pdf*

$$f_X(x) = \begin{cases} 2(x - 5.5) & \text{if } 5.5 \le x \le 6.5. \\ 0 & \text{otherwise.} \end{cases}$$

Similarly the arrival time Y of the woman has the *pdf*

$$f_Y(x) = \begin{cases} 3(x - 5.5)^2 & \text{if } 5.5 \le x \le 6.5. \\ 0 & \text{otherwise.} \end{cases}$$

Assume that the man and woman arrive independently of each other. If the first one to arrive at the restaurant will not wait for more than 15 minutes, find the probability that they will have the dinner together. (You might want to use the aid of a graph to identify the region of integration.)

8 Suppose that X and Y are independent random variables such that each have the possible values $1, 2, \ldots$ with

$$p_X(n) = p_Y(n) = (1-p)^{n-1} p, \quad n = 1, 2, \ldots..$$

where $0 < p < 1$. Find the probability mass function of $X + Y$.

9 Consider ten letters $A, B, C, D, E, F, G, H, I, J$ with their proper places identified respectively as $1, 2, \ldots., 10$. (For instance the proper place of the letter F is the sixth place.) If these ten letters are arranged in a row at random find the expected value and the variance of the number of these letters that will occupy their proper place.

TABLE 5.1 AREA $\Phi(x)$ UNDER THE STANDARD NORMAL CURVE TO THE LEFT OF x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998